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Neutrino oscillations from the splitting of Fermi points

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As was shown previously, oscillations of massless neutrinos may be due to the splitting of multiply degenerate Fermi points. In this Letter, we give the details and propose a three-flavor model of Fermi point splittings and neutrino mixings with only two free parameters. The model may explain recent experimental results from the K2K and KamLAND collaborations. There is also rough agreement with the data on atmospheric neutrinos (SuperK) and solar neutrinos (SNO), but further analysis is required. Most importantly, the *Ansatz* allows for relatively strong T–violating (CP–nonconserving) effects in the neutrino sector.

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1. INTRODUCTION

Neutrino oscillations are commonly associated with neutrino-mass differences; see, e.g., Refs. [1–3] for three reviews. But, the different propagation states might also be distinguished by some other characteristic. Two examples discussed in the literature are connected with violations of the equivalence principle [4, 5] and Lorentz invariance [6].

Lorentz noninvariance and CPT violation as emergent phenomena in a fermionic quantum vacuum have been discussed recently by Volovik and the present author [7]. It was noted that one possible consequence of the splitting of multiply degenerate Fermi points (to be defined later) could be neutrino oscillations. The question is whether or not this particular type of neutrino oscillation is compatible with the experimental data. If so, we may have an entirely new perspective on the neutrino sector.

The aim of this paper, then, is to provide an exploratory analysis of the experimental data on neutrino oscillations from the perspective suggested in Ref. [7]. In order to stress the difference with mass oscillations, we keep an eye open to the possibility that the experimental data could, after all, be compatible with relatively strong T (and CP?) violation in the neutrino sector.

The outline of this Letter is as follows. In Sec. 2, we discuss the case of two-flavor oscillations for massless left-handed neutrinos with Fermi point splitting. (A Fermi point is a point in three-momentum space at which the energy spectrum of the fermion considered has a zero.) In Sec. 3, we propose a simple three-flavor model of Fermi point splittings and neutrino mixings, which allows for strong T violation. The model has two free parameters, an energy scale B_0 and a phase δ, together with particular fixed values (equal or close to $\pi/4$) for the three mixing angles. In Sec. 4, we give the resulting expressions for the oscillation probabilities among the three flavors. In Sec. 5, we compare the results of the model with the experimental data on neutrino oscillations. The combined data from K2K and KamLAND (with input from SuperK) appear to favor T violation ($\sin\delta \neq 0$) over time-reversal invariance ($\sin\delta = 0$), but this remains to be confirmed. In Sec. 6, we present some concluding remarks.

2. TWO-FLAVOR NEUTRINO OSCILLATIONS

In the limit of vanishing Yukawa couplings, the Standard Model fermions are massless Weyl fermions and have the following dispersion law

$$\big(E_{a,f}(\mathbf{q})\big)^2 = \big|\, c\,\mathbf{q} + \mathbf{b}_a^{(f)} \big|^2 \,, \qquad (1)$$

for three-momentum $\mathbf{q}$ and with $\mathbf{b}_a^{(f)} = 0$ for the moment. Here, a labels the sixteen types of massless left-handed Weyl fermions in the Standard Model (with a hypothetical left-handed antineutrino included) and f distinguishes the three known fermion families.

The Weyl fermions of the original Standard Model have all $\mathbf{b}_a^{(f)}$ vanishing, which makes for a multiply degenerate Fermi point $\mathbf{q} = \mathbf{0}$. [Fermi points (gap nodes) $\mathbf{q}_n$ are points in three-dimensional momentum space at which the energy spectrum $E(\mathbf{q})$ of the fermionic quasiparticle has a zero, i.e., $E(\mathbf{q}_n) = 0$.] Nonzero parameters $\mathbf{b}_a^{(f)}$ in the dispersion law (1) describe the splitting of this multiply degenerate Fermi point. See Ref. [7] for a discussion of the physics that could be responsible for Fermi point splitting and Ref. [6] (and references therein) for a general discussion of Lorentz noninvariance.

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Now, consider the following pattern of spacelike splittings:

$$\mathbf{b}_a^{(f)} = Y_a \; \mathbf{b}^{(f)}\,, \;\; \text{for} \;\; a = 1, \ldots, 16, \;\; f = 1, 2, 3\,, \quad (2)$$

as given by Eq. (5.4) of Ref. [7], with a minor change of notation. Given the hypercharges Y_a of the Standard Model fermions, this pattern has only three unknowns, the vectors $\mathbf{b}^{(f)}$. The Fermi point splittings (2), for nonvanishing $\mathbf{b}^{(f)}$, violate CPT but the induced electromagnetic CPT–odd Chern–Simons-like term cancels out exactly, consistent with the tight experimental limits. Still, there may be other effects, for example neutrino oscillations (as long as the neutrinos are not too much affected by the mechanism of mass generation).

We therefore focus on massless left-handed neutrinos (hypercharge $Y_{\nu_L} = -1$) with Fermi point splittings (2). The dispersion law for a left-handed neutrino with three-momentum $\mathbf{q}$ is then given by

$$\big(E_{\nu_L,f}(\mathbf{q})\big)^2 = \big|\, c\,\mathbf{q} - \mathbf{b}^{(f)} \big|^2\,, \quad (3)$$

with $f = 1, 2, 3$, for three neutrinos.

In this section, we restrict our attention to oscillations between two flavors of neutrinos (see, e.g., Ref. [8] for further details). The mixing angle between the flavor eigenstates $|A\rangle$, $|B\rangle$ and propagation eigenstates $|1\rangle$, $|2\rangle$ will be denoted by θ_{mix}. These propagation states evolve differently as long as $\mathbf{b}^{(1)} \neq \mathbf{b}^{(2)}$ in the dispersion law (3).

For an initial neutrino with large enough momentum $|\mathbf{q}|$, the oscillation probability from flavor A to flavor B over a travel time t (travel distance $L \sim c\,t$) is readily calculated:

$$P(A \to B) \sim \sin^2\!\big(2\,\theta_{\text{mix}}\big)\; \sin^2\left(\frac{1}{2}\; \boldsymbol{\Delta b}^{(ff')} \cdot \widehat{\boldsymbol{q}} \;\; L/\hbar\, c\right), \quad (4)$$

with $\widehat{\boldsymbol{q}} \equiv \mathbf{q}/|\mathbf{q}|$ and $\boldsymbol{\Delta b}^{(ff')} \equiv \mathbf{b}^{(f)} - \mathbf{b}^{(f')}$, for $f = 1$ and $f' = 2$. The oscillation probability (4) is anisotropic and energy independent. The survival probability is given by $P(A \to A) = 1 - P(A \to B)$. Oscillation probabilities similar to Eq. (4) have been discussed, for example, in Sec. III B of Ref. [6].

Next, consider the following timelike splittings of Fermi points for the massless Standard Model fermions:

$$b_{0a}^{(f)} = Y_a \; b_0^{(f)}\,, \;\; \text{for} \;\; a = 1, \ldots, 16, \;\; f = 1, 2, 3\,, \quad (5)$$

as given by Eq. (6.5) of Ref. [7]. Again, the induced electromagnetic CPT–odd Chern–Simons-like term cancels out exactly. The dispersion law of a massless left-handed neutrino is now given by

$$\big(E_{\nu_L,f}(\mathbf{q})\big)^2 = \big(\, c\,|\mathbf{q}| - b_0^{(f)}\,\big)^2\,, \quad (6)$$

with $f = 1, 2, 3$, for three neutrinos. In order to stay to the usual neutrino phenomenology as close as possible, it is assumed in this paper (different from Ref. [7]) that $b_0^{(f)}$ in Eq. (6) is a CP–even parameter. The results of Sec. 5 are, however, independent of this assumption.

For a large enough momentum of the initial neutrino, there is again an energy-independent two-flavor oscillation probability,

$$P(A \to B) \sim \sin^2\!\big(2\,\theta_{\text{mix}}\big)\; \sin^2\left(\frac{1}{2}\; \Delta b_0^{(ff')} \;\; L/\hbar\, c\right), \quad (7)$$

with $\Delta b_0^{(ff')} \equiv b_0^{(f)} - b_0^{(f')}$, for $f = 1$ and $f' = 2$. The first-peak distance (half of the wavelength λ) occurs at

$$L^{\,\text{first-peak}} = \pi\,\hbar c \,/\, |\Delta b_0^{(ff')}| \approx 600\,\text{km}\; \left(\frac{10^{-12}\;\text{eV}}{|\Delta b_0^{(ff')}|}\right)\,. \quad (8)$$

For completeness, we also mention neutrino-mass oscillations [1–3,8] which are based on the Lorentz-invariant dispersion law

$$\big(E_{\nu,f}(\mathbf{q})\big)^2 = c^2\,|\mathbf{q}|^2 + m_f^2\,c^4 \sim \left(c\,|\mathbf{q}| + \frac{m_f^2\,c^3}{2|\mathbf{q}|}\right)^2, \quad (9)$$

for $|\mathbf{q}| \gg m_f$ and with $f = 1, 2$, for two neutrinos. The standard result,

$$P_{\text{mass-oscill}}(A \to B) \sim \sin^2\!\big(2\,\theta_{\text{mix}}\big)\; \sin^2\left(\frac{1}{2}\; \frac{m_f^2 - m_{f'}^2}{2\,E_\nu}\; L\,c^3/\hbar\right), \quad (10)$$

has, of course, the same basic structure as Eq. (7) but is now energy dependent. With $\Delta m^2 \equiv m_f^2 - m_{f'}^2$, the corresponding first-peak distance is

$$L^{\,\text{first-peak}}_{\,\text{mass-oscill}} = \pi\,\big(2E_\nu\,\hbar c\big) \,/\, \big(|\Delta m^2|\,c^4\big) \approx 600\,\text{km}\left(\frac{E_\nu}{\text{GeV}}\right)\left(\frac{2\times 10^{-3}\;\text{eV}^2/c^4}{|\Delta m^2|}\right), \quad (11)$$

for energies typical of "atmospheric neutrinos" (see Sec. 5.1).

3. THREE-FLAVOR SPLITTING AND TRI-MAXIMAL MIXING

For simplicity, we consider only the timelike splittings (5) in the rest of this Letter. Because the neutrino

oscillations are energy-independent, the analysis of the experimental data is entirely different from that of the usual mass oscillations.

To illustrate this point, we choose the following regular pattern for the Fermi point splittings of the three left-handed neutrinos with dispersion law (6):

$$b_0^{(f)} = f\, B_0\,, \quad \text{for} \quad f = 1, 2, 3\,. \tag{12}$$

In addition, we take "tri-maximal" values for the mixing angles which enter the unitary matrix V between flavor and propagation states (see Sec. 4):

$$\begin{aligned} \theta_{13} &= \arctan\sqrt{1/2} \approx \pi/5\,,\\ \theta_{21} &= \theta_{32} = \arctan 1 = \pi/4\,. \end{aligned} \tag{13}$$

This neutrino mixing matrix is parametrized as follows [2, 3]:

$$V \equiv \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{32} & s_{32} \\ 0 & -s_{32} & c_{32} \end{pmatrix} \cdot \begin{pmatrix} c_{13} & 0 & s_{13}\, e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}\, e^{i\delta} & 0 & c_{13} \end{pmatrix} \cdot \begin{pmatrix} c_{21} & s_{21} & 0 \\ -s_{21} & c_{21} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{14}$$

with two Majorana phases set to zero and the standard notation s_x and c_x for $\sin\theta_x$ and $\cos\theta_x$.

The particular values (13) maximize, for given phase δ, the T–violation (CP–nonconservation) measure [9]

$$J \equiv \frac{1}{8}\cos\theta_{13}\,\sin 2\theta_{13}\,\sin 2\theta_{21}\,\sin 2\theta_{32}\,\sin\delta\,. \tag{15}$$

This maximality condition on J is used only as a mathematical prescription to select unambiguously certain "large" values of the mixing angles.

At this moment, we do not want to speculate on possible explanations of the relations (12) and (13). There is a certain elegance to the model, with essentially two free parameters (B_0 and δ). In contrast, the standard interpretation of the experimental results on neutrino oscillations [2, 3] has three different neutrino masses, at least two different mixing angles, and one undetermined phase:

$$\begin{aligned} m_2^2 - m_1^2 &\approx 7\times 10^{-5}\ \mathrm{eV}^2/c^4,\\ |m_3^2 - m_2^2| &\approx 2\times 10^{-3}\ \mathrm{eV}^2/c^4,\\ \theta_{13} \approx 0\,,\ \ \theta_{21} \approx \theta_{32} &\approx \pi/4\,,\ \ \delta \in [-\pi, \pi]\,. \end{aligned} \tag{16}$$

These values would imply that T and CP violation in the neutrino sector are suppressed by a small value of the mixing angle θ_{13} [cf. Eq. (15)], which would not be the case for the *Ansatz* (12)–(14).

4. THREE-FLAVOR NEUTRINO OSCILLATIONS

Now define three flavor states $|A\rangle, |B\rangle, |C\rangle$ in terms of the propagation states $|1\rangle, |2\rangle, |3\rangle$, which have the dispersion law (6) with parameters $b_0^{(f)}$, $f = 1, 2, 3$, given by the pattern (12). In matrix form, the relation is

$$\begin{pmatrix} |A\rangle \\ |B\rangle \\ |C\rangle \end{pmatrix} = V^\star \begin{pmatrix} |1\rangle \\ |2\rangle \\ |3\rangle \end{pmatrix}, \tag{17}$$

where the star indicates complex conjugation. Here, we follow the conventions of Ref. [2], with the mixing matrix V defined by Eq. (14) for the particular values (13).

For a large enough momentum of the initial neutrino, the energy differences from Eq. (12) give the following oscillation probabilities:

$$\begin{aligned}
&P(A\to B) = (2/9)\ \sin^2(\Delta/2)\\
&\times\Big(4+\sqrt{3}\,c_\delta + \big(2+2\sqrt{3}\,c_\delta\big)\cos\Delta - 2\sqrt{3}\,s_\delta \sin\Delta\Big),\\
&P(A\to C) = (2/9)\ \sin^2(\Delta/2)\\
&\times\Big(4-\sqrt{3}\,c_\delta + \big(2-2\sqrt{3}\,c_\delta\big)\cos\Delta + 2\sqrt{3}\,s_\delta \sin\Delta\Big),\\
&P(A\to A) = 1 - P(A\to B) - P(A\to C)\,,\\[1em]
&P(B\to C) = (2/9)\ \sin^2(\Delta/2)\\
&\times\Big(13/4 - (3/4)\,\cos 2\delta + 2\,\cos\Delta - 2\sqrt{3}\,s_\delta\,\sin\Delta\Big),\\
&P(B\to A) = (2/9)\ \sin^2(\Delta/2)\\
&\times\Big(4+\sqrt{3}\,c_\delta + \big(2+2\sqrt{3}\,c_\delta\big)\cos\Delta + 2\sqrt{3}\,s_\delta \sin\Delta\Big),\\
&P(B\to B) = 1 - P(B\to C) - P(B\to A)\,,\\[1em]
&P(C\to A) = (2/9)\ \sin^2(\Delta/2)\\
&\times\Big(4-\sqrt{3}\,c_\delta + \big(2-2\sqrt{3}\,c_\delta\big)\cos\Delta - 2\sqrt{3}\,s_\delta \sin\Delta\Big),\\
&P(C\to B) = (2/9)\ \sin^2(\Delta/2)\\
&\times\Big(13/4 - (3/4)\,\cos 2\delta + 2\,\cos\Delta + 2\sqrt{3}\,s_\delta\,\sin\Delta\Big),\\
&P(C\to C) = 1 - P(C\to A) - P(C\to B)\,,
\end{aligned} \tag{18}$$

with the further definition

$$\Delta \equiv B_0\, t/\hbar \sim B_0\, L/(\hbar c) \tag{19}$$

and notation s_δ and c_δ for $\sin\delta$ and $\cos\delta$. For the antiparticle probabilities replace δ by $-\delta$ (assuming B_0 to be CP–even). The difference of the $P(X\to Y)$ and

$P(Y \to X)$ probabilities in Eq. (18), for $X \neq Y$ and $s_\delta \sin\Delta \neq 0$, implies T violation; cf. Ref. [3].

For later use, we also calculate the average probabilities $\langle P \rangle$, defined by integrating Δ over the interval $[0, 2\pi]$ with normalization factor $1/(2\pi)$:

$$\Big(\langle P(A \to B)\rangle\,,\,\langle P(A \to C)\rangle\,,\,\langle P(A \to A)\rangle\Big) =$$
$$\Big(1/3\,,\,1/3\,,\,1/3\Big)\,,$$
$$\Big(\langle P(B \to C)\rangle\,,\,\langle P(B \to A)\rangle\,,\,\langle P(B \to B)\rangle\Big) =$$
$$\Big(1/4 - (\cos 2\delta)/12\,,\,1/3\,,\,(5 + \cos 2\delta)/12\Big)\,,$$
$$\Big(\langle P(C \to A)\rangle\,,\,\langle P(C \to B)\rangle\,,\,\langle P(C \to C)\rangle\Big) =$$
$$\Big(1/3\,,\,1/4 - (\cos 2\delta)/12\,,\,(5 + \cos 2\delta)/12\Big)\,. \qquad (20)$$

These average probabilities are equal only for $\delta = \pm\,\pi/2$.

The identification of the states $|A\rangle, |B\rangle, |C\rangle$ with the usual neutrinos states $|\nu_e\rangle$, $|\nu_\mu\rangle$, $|\nu_\tau\rangle$ is left to experiment, which, after all, observes the electrons and the muons.

5. COMPARISON TO EXPERIMENT

In this section, we compare the model predictions (18) with two sets of data on neutrino oscillations, one from the SuperK and K2K experiments and the other from the KamLAND and SNO experiments. The LSND results are left out of consideration, as these have not been confirmed by another experiment. We refer to two recent reviews [2, 3] for further details and an extensive list of references.

5.1 SUPERK AND K2K

With neutrino energies in the GeV range, SuperK [10] discovered indirect evidence for $\nu_\mu \to \nu_x$ oscillations starting from a distance $L \approx 500$ km (corresponding to a zenith angle of approximately 90°). The same type of neutrino oscillations has also been inferred by K2K [11] at a distance $L = 250$ km. Both lengths are of the same order of magnitude as Eq. (8).

For a more precise analysis we turn to the K2K experiment. The crucial result is now that K2K [12] does *not* see $\nu_\mu \to \nu_e$ at an appreciable level for the length $L = 250$ km where the ν_μ flux is reduced by approximately 30 %. The quoted numbers of neutrino events are

$$\Big(N_{\nu_\mu}, N_{\nu_e}, N_{\nu_\tau}\Big)\Big|^{\text{K2K}}_{L=250\ \text{km}} \approx \big(56\,,\,1\,,\,23?\big)\,, \qquad (21)$$

where the number for N_{ν_τ} has been deduced from the expected number $N_{\nu_\mu} \approx 80 \pm 6$ without neutrino oscillations.

Taking the phase $\delta = \pi/4$, the probabilities calculated in Eq. (18) give a "best fit" for

$$80 \times \Big(P(C \to C), P(C \to A), P(C \to B)\Big)\Big|^{\delta=\pi/4}_{l=0.145} \sim$$
$$\big(56\,,\,2\,,\,22\big)\,, \qquad (22)$$

with the dimensionless length l defined by

$$2\pi\, l \equiv B_0\, L/(\hbar\, c) \sim \Delta\,. \qquad (23)$$

The model numbers (22) compare well with the "observed" numbers (21).

The comparison with the K2K experiment allows for the following tentative identification:

$$\Big(\ |A\rangle = |\nu_e\rangle\,,\ \ |B\rangle = |\nu_\tau\rangle\,,\ \ |C\rangle = |\nu_\mu\rangle\ \Big)\Big|^{\delta=\pi/4}\,, \qquad (24)$$

at least if δ is set to $\pi/4$ (see Sec. 5.2 for further discussion). With $L = 250$ km, we also have

$$B_0 \approx 0.145\,(hc)/(250\,\text{km}) \approx 7.2 \times 10^{-13}\ \text{eV} \qquad (25)$$

and a wavelength $\lambda \approx 1700$ km ($l = 1$). The statistical error on B_0 is estimated to be of the order of 10 %, as obtained by letting the N_{ν_τ} value in Eq. (21) range from 17 to 29 and finding the matching probabilities in the model.

The K2K experiment has also analyzed the spectrum of the reconstructed energies of the μ–type neutrinos. Given the large errors, the data points agree more or less with the shape expected from the Fermi-point-splitting mechanism (box histogram in Fig. 2 of Ref. [11]).

The production rates corresponding to Eq. (22) first have a peak for B–type neutrinos at $l \approx 0.3$ and then a peak for A–type neutrinos at $l \approx 0.6$, with the C–type rate reduced to under 20 % over the range $0.3 \lesssim l \lesssim 0.7$. For SuperK, the C–type (= μ–type?) atmospheric neutrinos would start being depressed at a length $L \approx 500$ km ($l \approx 0.3$) which is roughly what is observed at a zenith angle of 90°. With travel distances averaged over several thousand kilometers (corresponding to large enough zenith-angle intervals), the number of initial C–type neutrinos would be reduced significantly. According to Eq. (20) for $\delta = \pi/4$, an initial $2:1$ ratio of C–type to A–type neutrinos would be changed as follows:

$$\big(N_{C,\bar{C}} : N_{A,\bar{A}} : N_{B,\bar{B}}\big) = \big(120 : 60 : 0\big) \rightarrow$$
$$\big(50+20 : 40+20 : 30+20\big)\,. \tag{26}$$

Apparently, these averaged vacuum oscillations would keep the initial number of A–type (= e–type?) events unchanged and reduce the initial number of C–type (= μ–type?) events by 40 %, more or less as observed by SuperK [10].

Needless to say, a complete re-analysis of the SuperK data is required, if the neutrino energy is given by Eq. (6) instead of the Lorentz-invariant relation (9). The most important task would be to establish unambiguously whether or not the oscillation properties depend on the neutrino energy. (Fig. 4 of Ref. [10] is not really conclusive, because the data points can also be fitted by a smoothed steplike function, which drops from a constant value 1 for $L/E_\nu \lesssim 100$ km/GeV to a constant value 0.6 for $L/E_\nu \gtrsim 400$ km/GeV.)

5.2 KAMLAND AND SNO

With antineutrino energies in the MeV range, KamLAND [13] presented indirect evidence for $\bar{\nu}_e \to \bar{\nu}_x$ oscillations at a distance $L \approx 180$ km. The experiment quotes the following survival probability:

$$P(\bar{\nu}_e \to \bar{\nu}_e)\Big|^{\,\text{KamLAND}}_{\,L\approx 180\text{ km}} =$$
$$0.611 \pm 0.085\,(\text{stat}) \pm 0.041\,(\text{syst})\,. \tag{27}$$

The distance $L \approx 180$ km corresponds to $l \approx 0.104$, as defined by Eq. (23) for the tentative energy scale (25). From Eq. (18) specialized to $\delta = \pi/4$, the relevant probability for the identification (24) is

$$P\big(\bar{A} \to \bar{A}\big)\Big|^{\,\delta=\pi/4}_{\,l=0.104} \sim 0.74\,, \tag{28}$$

which is less than two standard deviations away from the experimental result (27); cf. Fig. 4 of Ref. [13]. Note that a 10 % error on the value of l translates into a 6 % error for the probability (28).

The KamLAND experiment has also analyzed the positron energy spectrum from the inverse β–decay used to detect the antineutrinos. The spectrum is reported to be consistent at the 53 % C.L. with the expectations from the Fermi-point-splitting mechanism (upper histogram in Fig. 5 of Ref. [13] multiplied by a factor 0.6).

Considering the oscillation probabilities (18) for only two values of the phase, $\delta = 0$ and $\delta = \pi/4$, the *combined* experiments of K2K, KamLAND, and SuperK appear to favor the nonzero value of δ. As an example of a disfavored identification (actually one of the best for $\delta = 0$), we list the following numbers:

$$80 \times \Big(P(A \to A), P(A \to C), P(A \to B)\Big)\Big|^{\,\delta=0}_{\,l=0.115} \sim$$
$$\big(56\,,\,2\,,\,22\big)\,, \tag{29}$$

$$P\big(\bar{C} \to \bar{C}\big)\Big|^{\,\delta=0}_{\,l=0.083} \sim 0.92\,. \tag{30}$$

The first set of numbers compares well with the K2K data (21) but the second number is rather far from the KamLAND result (27). The SuperK results, which indicate $\nu_\mu \to \nu_x$ wavelengths of at least 1000 km, also help to rule out certain other $\delta = 0$ identifications.

The model predictions for $\delta = -\pi/4$ and $\delta = \pm\pi/2$ have also been compared with the experimental data and "best fits" are found with numbers similar to those of Eqs. (29)–(30) or worse. The $\delta = \pi/4$ identification (24) seems to be preferred among the cases considered, at least for the Fermi point splittings (12) and mixing angles (13). Note that the δ range can be restricted to $[-\pi/2, \pi/2]$, since B and C switch roles in the probabilities (18) for $\delta \to \delta + \pi$.

The preliminary result for the T–violating phase is then

$$\delta \approx -3\pi/4 \;\text{ or }\; \pi/4\,, \tag{31}$$

with identifications $|A\rangle = |\nu_e\rangle$, $|B\rangle = |\nu_\mu\rangle$, $|C\rangle = |\nu_\tau\rangle$ for the case of $\delta = -3\pi/4$ and identifications (24) for $\delta = \pi/4$. A comprehensive statistical analysis remains to be performed in order to determine the error on these values for δ.

In contrast to KamLAND, the experiments of CHOOZ [14] and Palo Verde [15] failed to see evidence for $\bar{\nu}_e \to \bar{\nu}_x$ oscillations at $\ell \approx 1$ km. This would be consistent with the probabilities (18),

$$P(\bar{A} \to \bar{A}) = 1 - \mathrm{O}(\Delta_\ell^2\,)\,,\;\; P(\bar{A} \to \bar{B}) = \mathrm{O}(\Delta_\ell^2\,)\,,$$
$$P(\bar{A} \to \bar{C}) = \mathrm{O}(\Delta_\ell^2\,)\,, \tag{32}$$

for $|\bar{A}\rangle = |\bar{\nu}_e\rangle$ and with $\Delta_\ell \equiv B_0\,\ell/(\hbar c) \approx \ell/(270\text{km}) \ll 1$ for the energy scale (25). Remarkably, this general reduction of the off-diagonal oscillation probabilities does not require $\sin 2\theta_{13}$ to be close to zero as would be the case for mass oscillations (10)–(11) with $|m_1^2 - m_3^2| \approx 2 \times 10^{-3}\ \text{eV}^2/c^4$ and $E_\nu \approx 3$ MeV [14, 15].

As to solar neutrinos, SNO [16, 17] has definitely established flavor oscillations, with the initial e–type neutrinos distributed over the three flavors and their flux reduced to approximately 30 %. Vacuum oscillations of neutrinos with Fermi point splittings (12) and trimaximal mixing angles (13) have an average e–type survival probability of 1/3, according to Eqs. (20) and (24).

Matter effects can be expected to play a role because the matter-oscillation length scale $hc/(2\sqrt{2}\ G_F\, n_e)$ is approximately 100 km in the center of the Sun [3], which is definitely less than our length scale (8). But, in the end, matter effects may be rather unimportant if the vacuum mixing angles are close to $\pi/4$.

It is not clear how well our neutrinos with Fermi point splittings fit *all* the solar neutrino data from SNO and the other experiments [2, 3]. Obviously, a complete re-analysis of neutrino propagation in the Sun is required, if the vacuum dispersion law is given by Eq. (6).

There is also the possibility of further effects from small neutrino masses with their own matrix structure; cf. Ref. [6]. These small masses could affect flavor oscillations of solar neutrinos with relatively low energy ($E_\nu \lesssim 1$ MeV for $|\Delta m^2| \approx 10^{-6}$ eV$^2/c^4$), whereas oscillations of neutrinos with higher energy would be primarily determined by the Fermi point splittings (25).

6. CONCLUSION

The present Letter has shown that energy-independent neutrino oscillations from the timelike splitting of Fermi points [7] need not be in flagrant contradiction with the experimental data [10–17].

For the sake of argument, we have considered a simple model (12)–(14) with two free parameters, the energy scale B_0 and the phase δ. The mixing angles of this model are fixed to the values (13) by the condition that they maximize the function (15) for a given phase δ. It turns out that the model can more or less explain the results of the K2K and KamLAND experiments, with a fundamental energy scale B_0 of the order of 10^{-12} eV and a preference for a nonzero T–violating phase, $\sin^2\delta \approx 1/2$. (These numerical values are, of course, to be considered preliminary.) There is also rough agreement with the data on atmospheric neutrinos (SuperK) and solar neutrinos (SNO), but further analysis is needed.

The tentative conclusion is that the simple *Ansatz* (12)–(13) for the neutrino dispersion law (6) and mixing matrix (14) may be compatible with experiment. The model considered can, of course, be perturbed by changes in the energy scales and mixing angles and by the addition of small mass terms. More importantly, the *Ansatz* suggests an entirely new structure of the neutrino sector, with the possibility of relatively strong T (and CP?) violation.

We end this Letter with four general remarks. First, the spacelike splitting of Fermi points is a possibility not considered in detail here, as the phenomenology would certainly be more complicated due to the presence of anisotropies; cf. Eq. (4). Second, left-handed antineutrinos (with hypercharge $Y_{\bar{\nu}_L} = 0$) drop out for the patterns (2) and (5) but could perhaps play a role in further mass generation. Third, it remains to be seen how the Fermi point splittings of the massless (or nearly massless) neutrinos feed into the charged-lepton sector; cf. Ref. [18]. Fourth, theory and experiment need to elucidate the precise role, if any, of CP, T, and CPT violation in the neutrino sector.

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